values of the wedge angle α

$$\alpha,\ \text{deg.} = 190\ 210\ 230\ 250\ 270\ 290\ 310\ 330\ 350$$
$$\beta,\ \text{deg.} = 50\ \ 53\ \ 60\ \ 66\ \ 70\ \ 80\ \ 87\ \ 94\ \ 105$$

Note. It was pointed out by V.K. Vostrov that for $G^+(-1/_2)$ of /3/ the factor $\sqrt{2}(\sqrt{3}\sin\alpha\cos\alpha/2)^{-1}$ should be multiplied by the inverse expression $\sqrt{3/_2}\sin\alpha\cos\alpha/2$. The numerical factors in (4.9) and (4.12) will now become 0.058 and 0.28 respectively (compared with the previous 0.046 and 0.22).

REFERENCES

1. CHEREPANOV G.P., Mechanics of Brittle Fracture. Moscow, NAUKA, 1974.
2. KIPNIS L.A., Homogeneous problem for a wedge with a symmetric crack at the tip. PMM, Vol. 45, No.2, 1981.
3. CHEREPANOV G.P. Plastic rupture lines at the tip of a crack. PMM, Vol.40, No.4, 1976.

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ON A STAR-LIKE SYSTEM OF PROPAGATING DISLOCATION DISCONTINUITIES*

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An antiplane dynamic problem of a system of dislocation discontinuities propagating from the origin of coordinates and forming a star-like structure is considered. A displacement field is obtained and specific features of seismic radiation in the far zone are studied.

Let $2n$ dislocation discontinuities with uniform angular distribution (Fig.1) begin to propagate at the initial instant $t=0$ from the origin of a Cartesian system of coordinates *Oxy*, with constant velocity, in an isotropic elastic medium. We define the discontinuity kinematically, i.e. we specify at each point of the plane of discontinuity the magnitude and direction of the displacement jump vector at the discontinuity, depending on the coordinates and time. As was shown in /1–6/, the kinematic description of the discontinuities shows in many cases a number of preferences as compared with the dynamic method whereby the forces are defined at the discontinuity. An analogous problem for the cracks using the dynamic method of describing the discontinuities was studied in /7/.

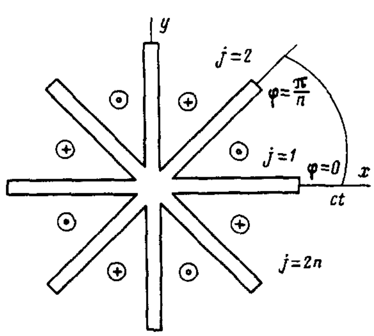

Fig.1

We shall assume that every single dislocation discontinuity is described by a symmetric (about the plane of discontinuity) homogeneous function of zero dimension $f(\rho/t)$. We denote by σ_{xz} and σ_{yz} the stress tensor components and by w the unique non-zero displacement vector component satisfying the wave equation

$$\frac{\partial^2 w}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial w}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2 w}{\partial\varphi^2}=\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} \tag{1}$$
$$\left(\rho=\sqrt{x^2+y^2},\ \varphi=\operatorname{arctg}\frac{y}{x}\right)$$

where ρ, φ are polar coordinates and c is the velocity of transverse waves. The boundary conditions are

$$\left.\begin{matrix}[w]=f(\rho/t), & \rho\leqslant vt\\ [w]=0, & \rho>vt\end{matrix}\right\}\varphi=0,\frac{\pi}{n};\quad n=1,2,3\ldots \tag{2}$$

Thus we must find a solution of problem (1), (2) belonging to the class of selfsimilar problems with the selfsimilarity index (0,0). We use the Smirnov-Sobolev method /3/ of the functionally invariant solutions, and the general approach employed in solving such problems /9/, enabling us to reduce the selfsimilar problems of the dynamic theory of elasticity to the boudary value problems of the theory of analytic functions.

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The general solution of problem (1), (2) can be written in terms of the analytic function W of complex variable

$$w = \operatorname{Re} W(z), \quad \sigma_{xz} = \mu \operatorname{Re} W'(z)\frac{\partial z}{\partial x}, \quad \sigma_{yz} = \mu \operatorname{Re} W'(z)\frac{\partial z}{\partial y} \tag{3}$$

Since the problem is symmetrical, we shall restrict ourselves to considering the sector $0 < \rho < ct$, $0 \leqslant \varphi \leqslant \pi/n$. It should be noted that, depending on the direction of the displacement jump at the discontinuity, two different types of deformation are possible within the sector $0 \leqslant \varphi \leqslant \pi/n$. In the first case we shall have "torsion of the angle" when the displacement will have different signs at $\varphi = 0$ and $\varphi = \pi/n$, and in the second case we shall have "bending of the angle" in which case the signs will be the same.

Taking into account the selfsimilarity of the problem and the properties of the functionally invariant solutions /8, 9/, we can reduce the equation of motion and the boundary conditions, in order to obtain w, to the form

$$\Delta w(r,\varphi) = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2} = 0 \tag{4}$$

$$\left.\begin{aligned} &w = {}^1\!/_2 f[\xi(r_1)], \quad 0 < r_1 < R \\ &w = 0, \qquad\qquad R < r_1 < 1 \end{aligned}\right\} \varphi = 0, \frac{\pi}{n} \tag{5}$$

$$w = 0, \quad r_1 = 1, \quad 0 \leqslant \varphi \leqslant \frac{\pi}{n}$$

$$\left(r_1 = \frac{1-\sqrt{1-\xi^2}}{\xi}, \ \xi = \frac{\rho}{ct}; \ R = \frac{1-\sqrt{1-\gamma^2}}{\gamma}, \ \gamma = \frac{v}{c}\right)$$

The sector $0 \leqslant \varphi \leqslant \pi/n$ of the circle of infinite radius will now become the sector $0 \leqslant \varphi \leqslant \pi/n$ of the unit circle, and the coordinates of the complex plane z_1 will be connected with the polar coordinates by the relation

$$z_1 = r_1 e^{i\varphi_1}, \quad \varphi_1 = \varphi$$

Using the conformal transformation

$$z = 2\,[z_1{}^n + z_1{}^{-n}]^{-1}$$

we map the interior of the sector $0 \leqslant \varphi \leqslant \pi/n$ of the unit circle onto the upper half-plane of the region z. The ends of the discontinuities moving outwards become the points $\pm x_1$ where $x_1 = 2\,[R^n + R^{-n}]^{-1}$. The modulus and argument of the complex variable z is expressed in terms of ρ and φ as follows:

$$|z| = \frac{2r_1{}^n\,[(r_1^{2n}+1)^2\cos^2 n\varphi + (1-r_1^{2n})^2\sin^2 n\varphi]^{1/2}}{r_1^{4n} + 2r_1^{2n}\cos 2n\varphi + 1}$$

$$\operatorname{Arg} z = \operatorname{arctg}\frac{(1-r_1^{2n})\operatorname{tg} n\varphi}{r_1^{2n}+1}$$

Then we can write the boundary conditions (5) in the form

$$\begin{aligned} &\operatorname{Re} W(z) = {}^1\!/_2 f, \quad |\operatorname{Re} z| < x_1, \ \operatorname{Im} z = 0 \\ &\operatorname{Re} W(z) = 0, \quad |\operatorname{Re} z| > x_1, \ \operatorname{Im} z = 0 \end{aligned} \tag{6}$$

The solution of the boundary value problem (6) can be written in terms of the Schwartz integral /10/ as follows:

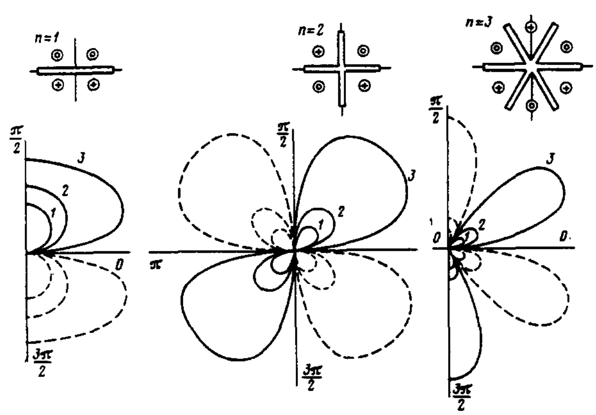

Fig.2

$$W(z) = \frac{1}{2\pi i} \int_{-x_1}^{x_1} \frac{f(t)\,dt}{t-z} + ic_0 \tag{7}$$

Knowing the function $W(z)$, we can use (3) to write the stress and displacement components over the whole region.

Let us consider in more detail the case when $f(\rho/t) = b = \mathrm{const}$. In this case, using (7) we obtain the following expressions for the solution function $W(z)$:

$$W(z) = \frac{bi}{2\pi} \ln \frac{z^2 - x_1^2}{z^2} \quad \text{(torsion of the angle)} \tag{8}$$

$$W(z) = \frac{bi}{2\pi} \ln \frac{z - x_1}{z + x_1} \quad \text{(bending of the angle)} \tag{9}$$

Let us analyze the dynamic displacement field for the case of bending of the angle. Substituting (9) into (3) and separating the real and imaginary parts, we obtain the following expressions for the displacement field:

$$w(\rho, \varphi, t) = -\frac{b}{2\pi} \operatorname{arctg} \frac{2p_k n_k \sin n\varphi}{m_k^2 \cos^2 n\varphi + p_k^2 \sin^2 n\varphi - n_k^2}$$

$$p_k = r_1^n (1 - r_1^{2n}),\ m_k = R^n (r_1^{2n} + 1)$$

$$n_k = R^n (r_1^{4n} + 2r_1^{2n} \cos 2n\varphi + 1)(R^{2n} + 1)^{-1}$$

For the first entry of the wave to the observer point, i.e. as $r_1 \to 1$, we obtain the following near-frontal asymptotics:

$$w(\rho, \varphi, t) = \frac{b}{2\pi} \frac{4nR^n (R^{2n} + 1)\sin n\varphi}{(R^{2n} + 1)^2 - 4R^{2n} \cos^2 n\varphi} \sqrt{\frac{ct}{\rho} - 1} \tag{10}$$

The analysis of the near-frontal asymptotics is given much attention in connection with the study of the direction in which the seismic energy radiates, the energy originating at tectonic earthquake foci and connected with the formation and rapid growth of the shear discontinuities within the earth crust. Fig.2 shows the diagrams of directions of radiated seismic energy caused by a system of $2n$ shear discontinuities propagating outwards for $n = 1, 2, 3$. Every point in these diagrams is defined by a radius vector whose modulus is proportional to the magnitude of the displacement at that point, and the direction coincides with the direction from the centre of the system of the discontinuities towards the point in question. The solid lines represent the parts of the space in which the displacements have positive signs, and the dashed lines those with negative sign (since we have symmetry with respect to the vertical, only the right hand sides of the diagrams are shown for $n = 1,3$). We see that the magnitude of the displacements increases as the velocity of propagation of the discontinuities increases (lines 1, 2, 3 have the corresponding values of $\gamma = 0.4;\ 0.6;\ 0.9$). The general pattern of directions is however preserved. The number of nodal planes from which there are no displacements and during the passage through which the displacements change their sign, is equal to n, and the nodal planes themselves coincide with the planes of discontinuity. In the case of $n = 2$ the diagram of radiation directions shows a clearcut distribution of signs over the quadrants analogous to that observed when analyzing the actual seismograms of large-scale tectonic earthquakes. This leads us to the assumption that in specific cases the focus zone of the tectonic earthquake can be modelled with the help of a system comprising four antiplane shear discontinuities propagating outwards.

REFERENCES

1. BYKOVTSEV A.S., Displacement field generated by a repeatedly propagating and halting slip plane. In a book: Mechanics of Deformable Media. Kuibyshevskii un-t. 1978.
2. BYKOVTSEV A.S. and CHEREPANOV G.P., On the simulation of a seismic centre. PMM Vol.44, No. 3, 1980.
3. BYKOVTSEV A.S. and CHEREPANOV G.P., On the limiting velocity rectilinear propagation of a dislocation shear discontinuity. Dokl. Akad. Nauk UzbSSR, No.3, 1980.
4. BYKOVTSEV A.S. and CHEREPANOV G.P., Analog of the Griffith problem in the theory of dislocation jumps. Dokl. Akad. Nauk UzbSSR, No.1, 1981.
5. BYKOVTSEV A.S. and CHEREPANOV G.P., On the steady motion of a dislocation jump. Izv. Akad. Nauk UzbSSR, ser. tekh. n. No.3, 1981.
6. BYKOVTSEV A.S., On the conditions of onset of motion of two collinear dislocation jumps. PMM Vol.47, No.4, 1983.
7. MARTYNIUK P.A. and SHER E.N., On selfsimilar development of a star-like system of cracks under the conditions of antiplane deformation. In book: Problems of Mechanism or rock fracture. Novosibirsk, Izd-e In-ta gorn. dela SO Akad. Nauk SSSR, 1976.

8. SOBOLEV S.L., Certain problems in the theory of wave propagation. In book by Frank F. and Mises R., Differential and Integral Equations of Mathematical Physics. Leningrad-Moscow, ONTI, 1937.
9. AFANAS'EV E.F. and CHEREPANOV G.P., Certain dynamic problems of the theory of elasticity. PMM Vol.37, No.4, 1973.
10. LAVRENT'EV M.A. and SHABAT B.V., Methods of the Theory of Functions of a Complex Variable. Moscow, Fizmatgiz. 1958.

Translated by L.K.

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INTEGRAL FORM OF THE GENERAL SOLUTION OF EQUATIONS OF STEADY-STATE THERMOELASTICITY*

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A new integral formula is obtained for solving the equation of steady-state thermoelasticity in a three-dimensional region, differing from the well-known formula /1/ in containing no volume integral. A similar formula is encountered in the case of a two-dimensional region, and its use in constructing the integral equation for boundary value problems is suggested. The fact that there are no volume integrals in the integral equations facilitates their numerical solution. If the temperature is represented by Green's formula in terms of the Newtonian potentials of the single and double layer, and the mass force is conservative, then, as shown below, the volume integrals will also be transformed into surface integrals over the boundary surface. The resulting formula however is less suitable for the numerical solution of boundary value problems as it contains a large number of integrals with different kernels.

1. The differential equations of equilibrium of a thermoelastic medium written in terms of the displacements u_i $(i = 1, 2, 3)$ have the form

$$\mu\Delta u_i + (\mu + \lambda)\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \frac{\alpha E}{1-2\nu}\frac{\partial T}{\partial x_i} - K_i \tag{1.1}$$

Here μ and λ are Lamé constants, E and ν is Young's modulus and Poisson's ratio, α is the coefficient of linear thermal expansion and K_i is the mass force density vector. The temperature T is sought in the form of the solution of an independent boundary value problem for the Laplace equation, and is assumed known. We write the solution of (1.1) in the form /1/

$$u_j(x) = \int_S [p_i(y)\,u_{ij}(x, y) - u_i(y)\,p_{ij}(x, y)]\,dS_y + \tag{1.2}$$

$$\int_D K_i(y)\,u_{ij}(x, y)\,dy + \frac{\alpha E}{1-2\nu}\int_D T(y)\,\theta_j(x, y)\,dy$$

$$u_{ij}(x, y) = \frac{(3-4\nu)\,\delta_{ij} + \beta_i\beta_j}{16\pi\mu(1-\nu)\,r}, \quad \theta_j(x, y) = \frac{\partial u_{ij}}{\partial y_j}$$

$$p_{ij}(x, y) = \frac{1-2\nu}{8\pi(1-\nu)\,r^2}\left(n_i\beta_j - n_j\beta_i - \delta_{ij}\cos\varphi - \frac{3\beta_i\beta_j\cos\varphi}{1-2\nu}\right)$$

Here $x(x_1, x_2, x_3)$ and $y(y_1, y_2, y_3)$ denote arbitrary points of the closed region $\bar{D}$, β_i are the direction cosines of the vector $r_i = y_i - x_i$ (r is its modulus), n_i are the direction cosines of the outward normal to the boundary S, φ is the angle between the vector with components r_i and the normal, $p_i(y)$ are the stress vector components on the surface with normal $\{n_i\}$, $dy = dy_1dy_2dy_3$ is the volume element of the region D, δ_{ij} is the Kronecker delta. We will write the Green identity for the function T and $\partial r/\partial y_j$ as follows:

$$\int_D \left[T\Delta\left(\frac{\partial r}{\partial y_j}\right) - (\Delta T)\frac{\partial r}{\partial y_j}\right]dy = \frac{\partial}{\partial x_j}\int_S\left(\frac{\partial T}{\partial n}r - T\frac{dr}{dn}\right)dS \tag{1.3}$$

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